Math 2940 Quiz 5 Solutions

November 14th, 2019

Name:

NetID:

Find both the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

Solution: To compute the eigenvalues, we want to find the roots of the characteristic equation:

$$\det(A - \lambda I) = \det\left(\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 2-\lambda & 3 \\ 4 & 1-\lambda \end{bmatrix}\right) = 0$$

$$(2-\lambda)(1-\lambda) - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$(\lambda - 5)(\lambda + 2) = 0$$

So $\boxed{\text{The eigenvalues are } \lambda = 5 \text{ and } \lambda = -2.}$

For the eigenvector associated with $\lambda = 5$, we want to find the nullspace of $A - \lambda I$:

$$A - 5I = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix} \xrightarrow{\text{R2 - R1}} \begin{bmatrix} -3 & 3 \\ 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{3}\cdot\text{ R1}} \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

Therefore the nullspace is described by the equation $-x_1 + x_2 = 0$. Plugging in $x_1 = 1$ we get that $x_2 = 1$, so

$$\boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ is an eigenvector with corresponding eigenvalue } \lambda = 5.}$$

For the eigenvector associated with $\lambda = -2$, we want to find the nullspace of $A - \lambda I$:

$$A + 2I = \begin{bmatrix} 4 & 3 \\ 4 & 3 \end{bmatrix} \xrightarrow{\text{R2 - R1}} \begin{bmatrix} 4 & 3 \\ 0 & 0 \end{bmatrix}$$

Therefore the nullspace is described by the equation $4x_1 + 3x_2 = 0$. Plugging in $x_1 = -3$ we get that $x_2 = 4$, so

$$\boxed{\begin{bmatrix} -3 \\ 4 \end{bmatrix} \text{ is an eigenvector with corresponding eigenvalue } \lambda = -2.}$$