Math 2940 Quiz 3
Solutions
October 17th, 2019

Section 208

Name:

NetID:

Find an explicit description of Nul A by listing the vectors that span the null space.

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -9 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution: To find the nullspace, we want to describe the solution to the homogeneous problem $A\boldsymbol{x} = \boldsymbol{0}$ in parametric vector form. Writing out $A\boldsymbol{x} = \boldsymbol{0}$:

$$\begin{aligned} x_1 - 2x_2 \quad + 4x_4 \quad &= 0 \\ x_3 - 9x_4 \quad &= 0 \\ x_5 &= 0 \end{aligned}$$

we can move all the free variables to the right hand side to get:

$$\begin{aligned} x_1 &= 2x_2 - 4x_4 \\ x_3 &= 9x_4 \\ x_5 &= 0 \end{aligned}$$

which we can write in parametric vector form as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_2 - 4x_4 \\ x_2 \\ 9x_4 \\ x_4 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - x_4 \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix}$$

Here we can see that the two basis vectors are $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix}$, so

$$\text{Nul } A = \text{Span} \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix} \right\}$$