Name:

NetID:

Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$.

Determine if $\boldsymbol{w}$ is in Col A. Is $\boldsymbol{w}$ in Nul A?

Solution:

To determine if $\boldsymbol{w}$ is in Col A, we want to see if there is a vector $\boldsymbol{x}$ such that $A\boldsymbol{x} = \boldsymbol{w}$. This is equivalent to the augmented matrix

$$\begin{bmatrix} -8 & -2 & -9 & | & 2 \\ 6 & 4 & 8 & | & 1 \\ 4 & 0 & 4 & | & -2 \end{bmatrix}$$

Row reducing this matrix,

$$\sim \begin{bmatrix} 4 & 0 & 4 & | & -2 \\ 6 & 4 & 8 & | & 1 \\ -8 & -2 & -9 & | & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 0 & 4 & | & -2 \\ 0 & 4 & 2 & | & 4 \\ 0 & -2 & -1 & | & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 0 & 4 & | & -2 \\ 0 & 4 & 2 & | & 4 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

From this we see that the system is *consistent*, so therefore $\boxed{\boldsymbol{w} \text{ is in Col } A.}$

To determine if $\boldsymbol{w}$ is in Nul A, we just compute $A\boldsymbol{w}$ and check if it equals the zero vector.

$$A\boldsymbol{w} = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} (2)(-8) + (1)(-2) + (-2)(-9) \\ (2)(6) + (4)(1) + (8)(-2) \\ (2)(4) + (1)(0) + (4)(-2) \end{bmatrix} = \begin{bmatrix} -16 - 2 + 18 \\ 12 + 4 - 16 \\ 8 - 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $A\boldsymbol{w} = \boldsymbol{0}$, we see that $\boxed{\boldsymbol{w} \text{ is in Nul } A.}$