Leibniz's formula general case

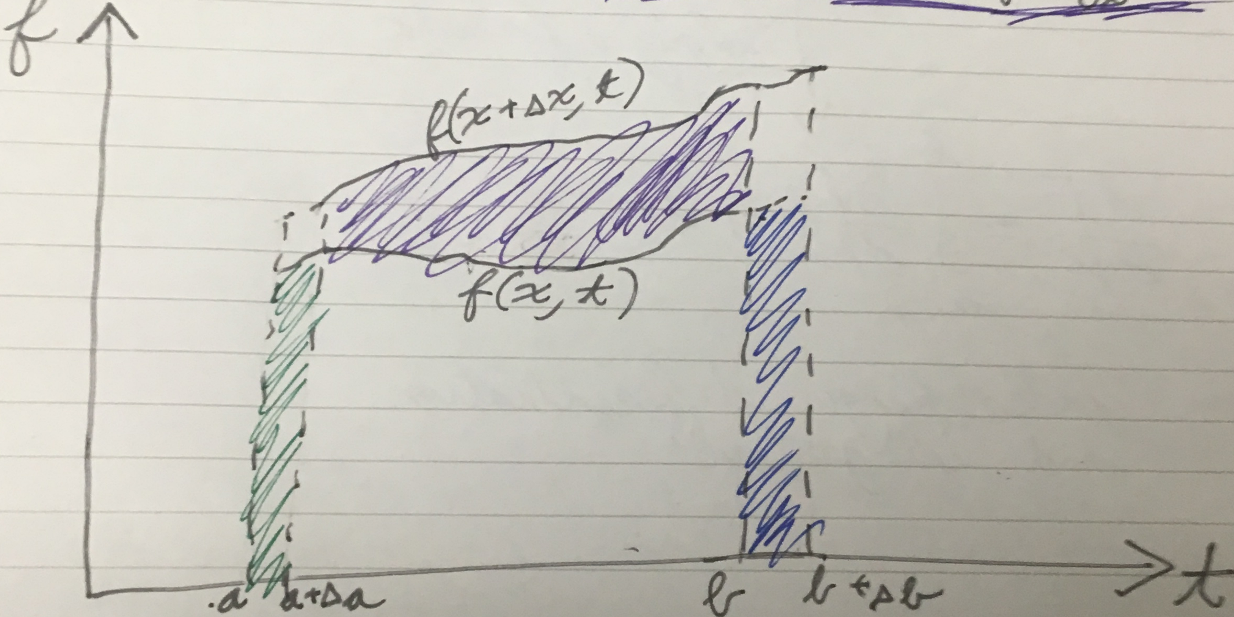
$$F(x) := \int_{a(x)}^{b(x)} f(x, t)\, dt$$

as x changes to $x + \Delta x$:

① lower limit becomes $a(x+\Delta x) = a + \Delta a \approx a + \frac{da}{dx} \cdot \Delta x$

② upper limit becomes $b(x+\Delta x) = b + \Delta b \approx b + \frac{db}{dx} \cdot \Delta x$

③ integrand changes to $f(x+\Delta x, t) = f + \frac{\partial f}{\partial x} \cdot \Delta x$



$$F(x+\Delta x) = \int_{a+\Delta a}^{b+\Delta b} f(x+\Delta x, t)\, dt = \int_{a+\Delta a}^{a} f(x+\Delta x, t)\, dt + \int_{a}^{b} f(x+\Delta x, t)\, dt + \int_{b}^{b+\Delta b} f(x+\Delta x, t)\, dt$$

$$F(x+\Delta x) \approx -\Delta a \cdot f(a(x), t) + \int_{a}^{b} \left(f + \frac{\partial f}{\partial x} \Delta x\right) dt + \Delta b \cdot f(b(x), t)$$

$$\frac{F(x+\Delta x) - F(x)}{\Delta x} \approx -\frac{\Delta a}{\Delta x} \cdot f(a(x), t) + \int_{a}^{b} \frac{\partial f}{\partial x}\, dt + \frac{\Delta b}{\Delta x} \cdot f(b(x), t)$$

$$\boxed{\frac{dF}{dx} = -\frac{da}{dx} \cdot f(a(x), t) + \frac{db}{dt} \cdot f(b(x), t) + \int_{a}^{b} \frac{\partial f}{\partial x}\, dt}$$